17th Swiss Geoscience Meeting

Continuous time random walk model for non-uniform bed-load transport with heavy-tailed hop distances and waiting times

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Publication information: Li Z P, Sun H G, Zhang Y, et al. Journal of Hydrology, 2019, 578: 124057.

1. Introduction

Motivation

Bed-load transport along widely graded river-beds typically exhibits anomalous dynamics, whose efficient characterization may require parsimonious stochastic models with pre-defined statistics involving the waiting time and hop distance distributions for sediment particles. Bridge between the individual particle motion and the macroscopic phenomena.

Objective

Characterizing the nature of time-dependent anomalous dispersion behavior in bed-load transport and interpret the physical mechanisms of widely graded bed-load particles.

Advantages

- **Providing a convenient model to simulate bedload transport.**
- **Describing the transport behavior from anomalous to normal, and well character the physical mechanism of anomalous behavior.**

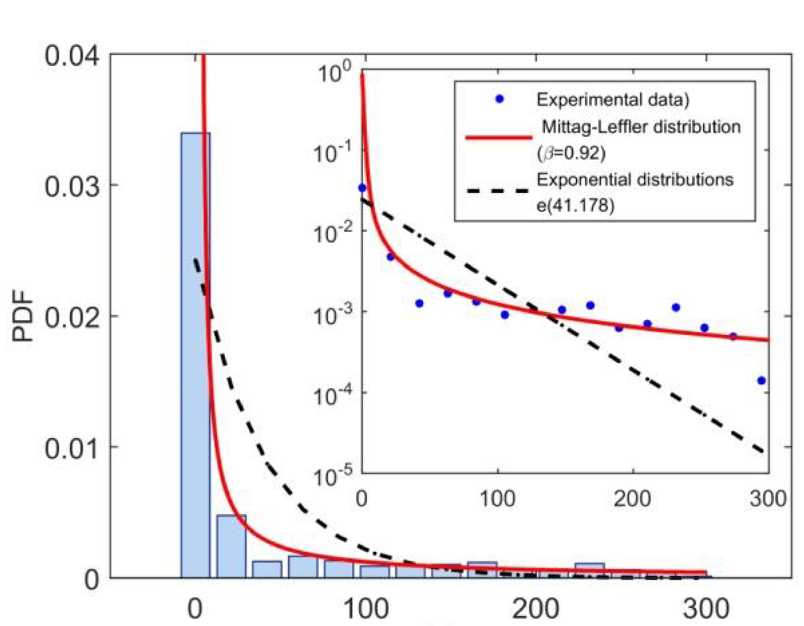
2. Theory and Methodology

Decoupled CTRW master equation

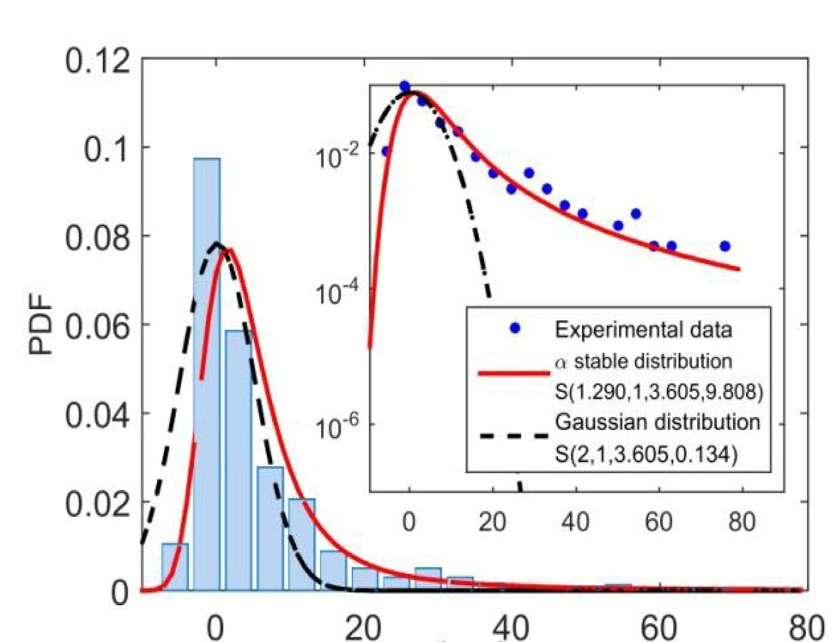
$$p(x,t)=\delta(x)\Psi(t)+\int_{-\infty}^{+\infty}d\xi\int_0^t d\tau\lambda(x)\psi(\tau)p(x-\xi,t-\tau)$$

$$p(k,s)=\frac{\tilde{\Psi}(s)}{1-\hat{\lambda}(k)\tilde{\psi}(s)}$$

Experimental data for waiting time and jump step distribution



Mittag-Leffler distribution



Levy alpha-stable distribution

For M-L distribution and Levy distribution

$$\tilde{\psi}(s)=\frac{1}{1+(\gamma_t s)^{\beta}} \qquad 1-\hat{\lambda}(k)\sim\gamma_x^{\alpha}|k|^{\alpha} \qquad \tilde{\Psi}(s)=\frac{1-\tilde{\psi}(s)}{s}=\frac{\gamma_t^{\beta}s^{\beta-1}}{1+(\gamma_t s)^{\beta}}$$

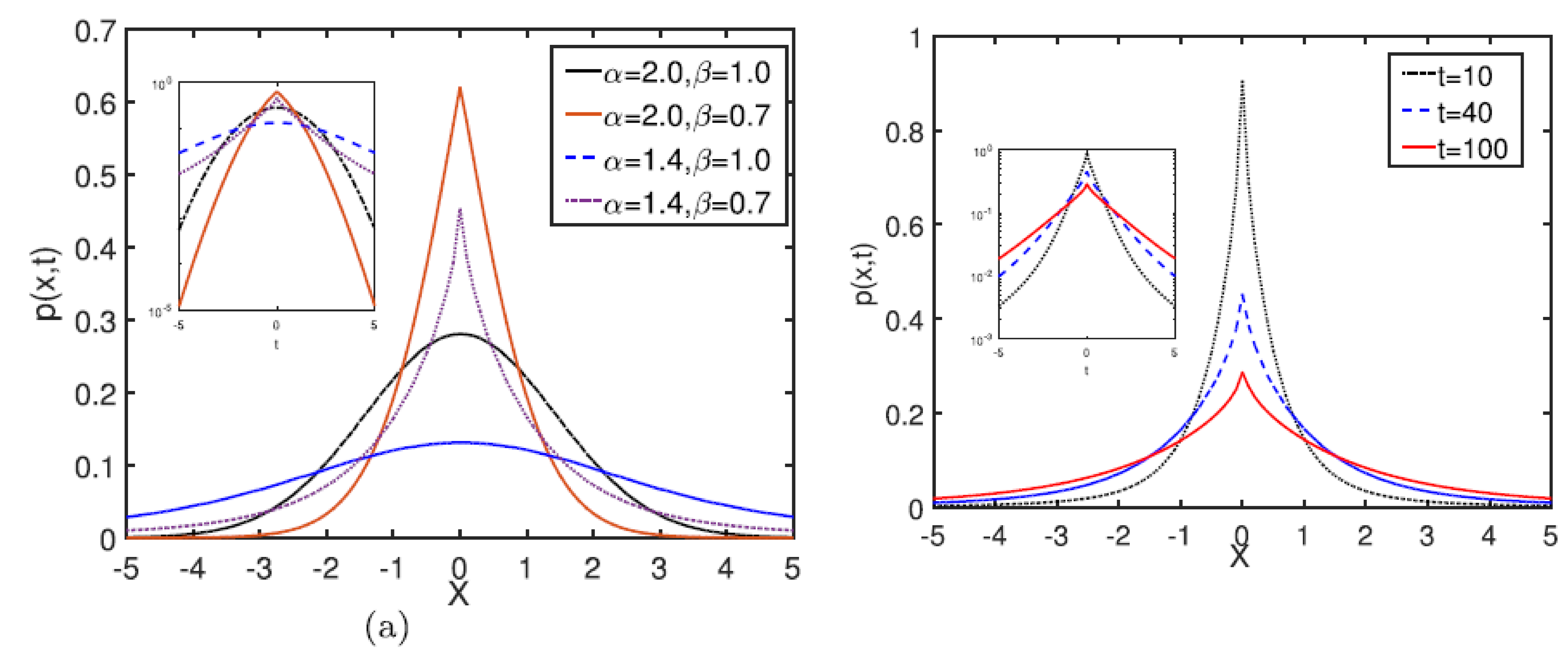
$${}_0D_t^{\beta}p(x,t)+D\cdot\frac{\partial^{\alpha}}{\partial x^{\alpha}}p(x,t)=\frac{t^{-\beta}}{\Gamma(1-\beta)}\delta(x)$$

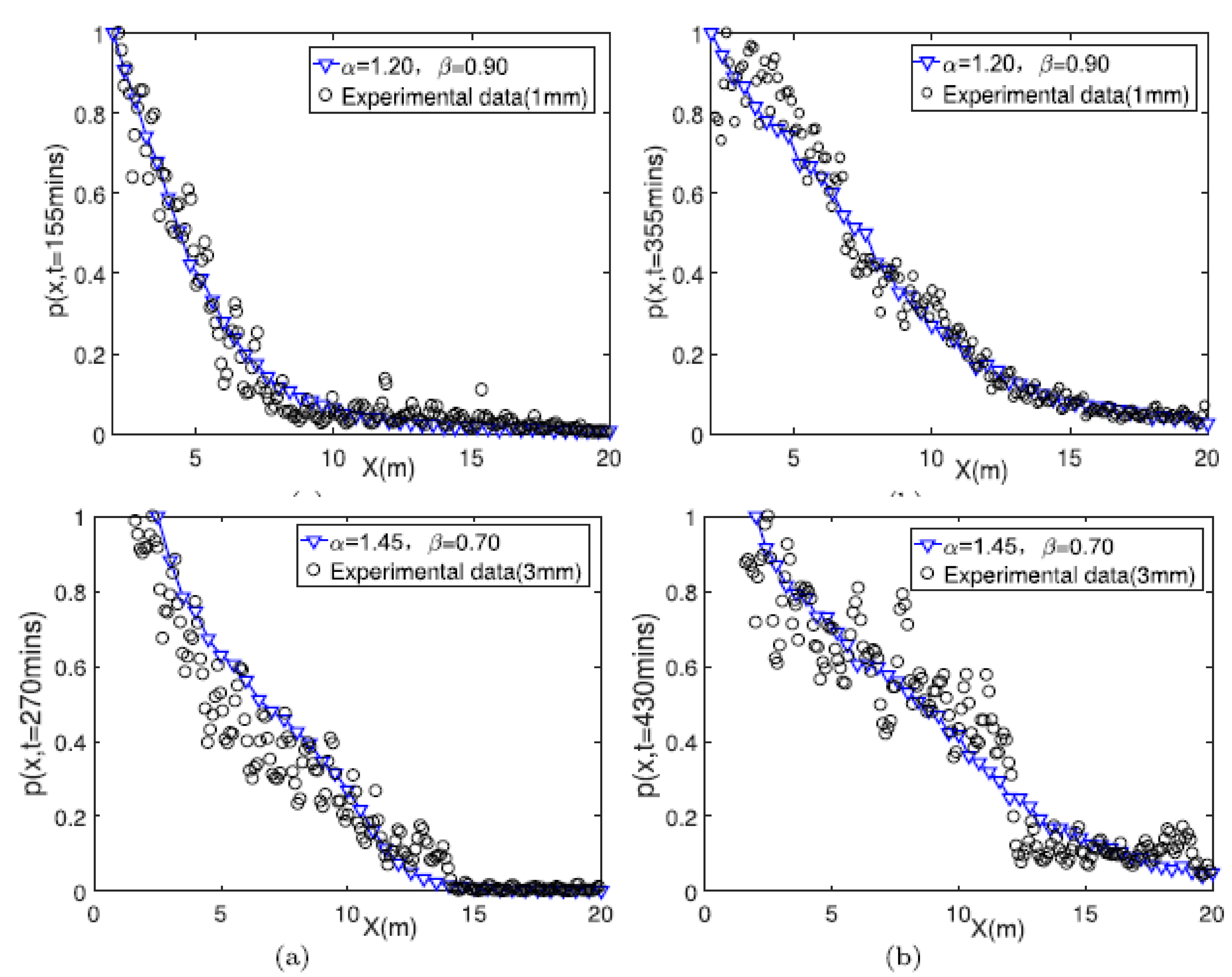
$${}_0D_t^{\beta}p(x,t)=L_s^{-1}[s^{\beta}\tilde{p}(x,s)](t)$$

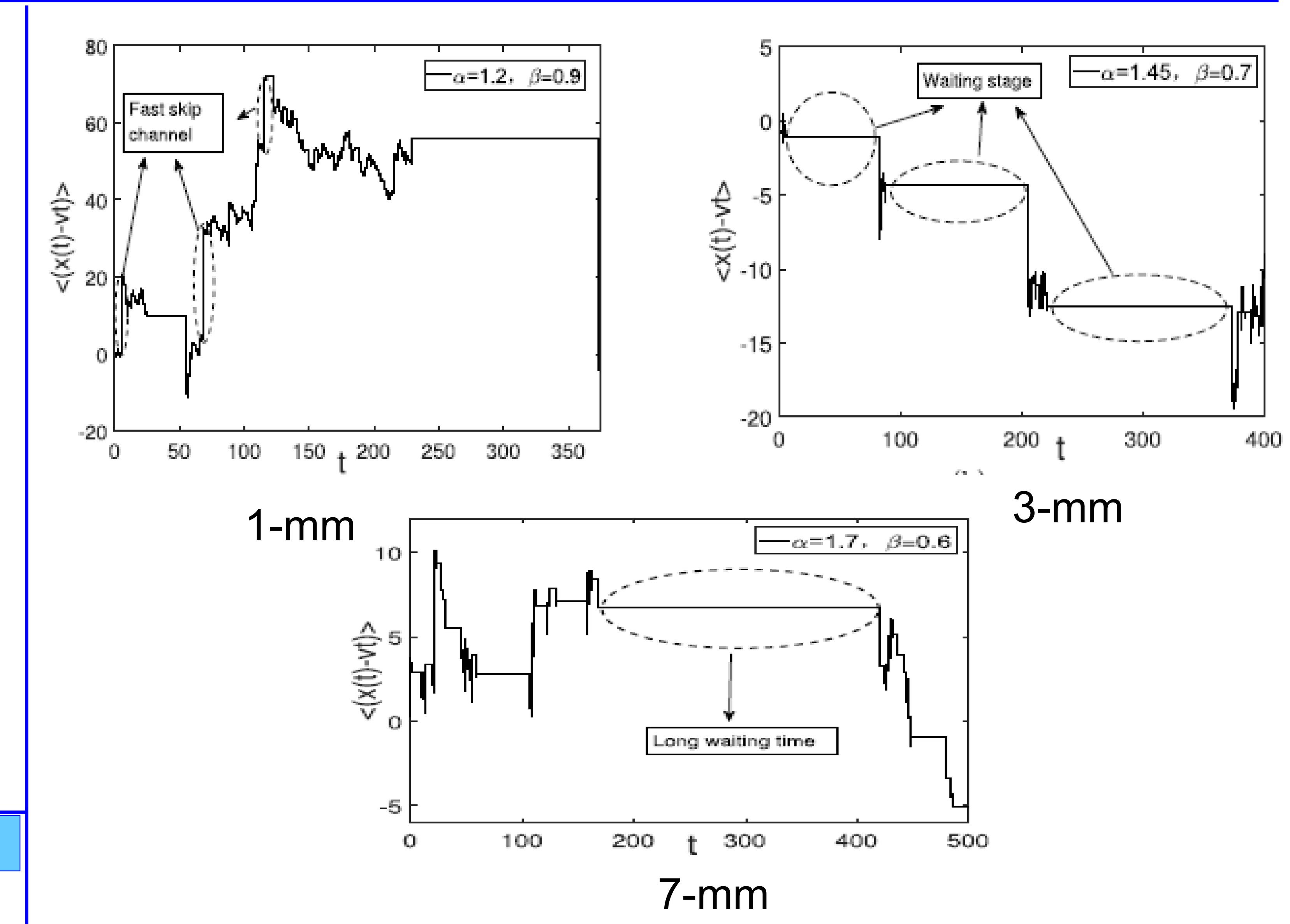
$$p(x,t)=D^{\beta}t^{-\beta/\alpha}Q(xD^{\beta}t^{-\beta/\alpha};\alpha,\beta)$$

$$Q(\xi;\alpha,\beta)=F_k^{-1}[E_{\beta}(-|k|^{\alpha})](\xi)$$

3. Numerical simulation







4. Discussions

- **The riverbed structures can accelerate the small particles while blocking the large particles**
- **The super-dispersion weakens with an increasing particle size, and super dispersion for bed-load with the instantaneous source is weaker than that of continuous source.**

5. Conclusions

- **Applications show that the CTRW model with heavy-tailed hop distances and waiting times can efficiently characterize the observed transport behavior for non-uniform bed-load sediment under different source load conditions.**
- **The flume experimental data and the CTRW model indicate that anomalous dispersion of bed-load is sensitive to the size of particles. Under the condition of low intensity sediment transport, the fine particles exhibit stronger super-dispersion, while the coarse particles show stronger sub-dispersion since they can be trapped for a relatively longer time on the sand bed.**
- **The particle-formed cluster and the fast channel structure on the river bed may be the main factor leading to the anomalous dispersion behavior of bed-load. Some particles are blocked/trapped by clusters, others can enter the flow accelerating belts, which lead to the broad distribution of random waiting times and hop distances.**

6. Acknowledgements

This work was funded by the National Key R&D Program of China (2017YFC0405203), the National Natural Science Foundation of China under Grant Nos. 11572112, 41931292, 11811530069 and 11772121, and the Fundamental Research Funds for the Central Universities under Grant Nos. 2015B03814 and 2017B21614. Renat T. Sibatov thanks the Russian Foundation for Basic Research (18-51-53018).

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